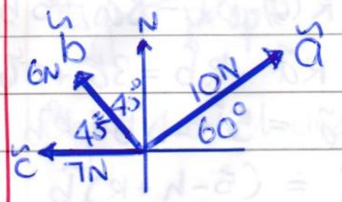
Specialist Mathematics Unit 1: **Chapter 4**

Ex 4A.

Q1 → 4

* I have used component vectors instead of scale drawings

1.

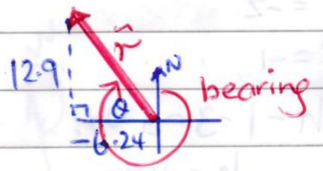


$\tilde{a} \Rightarrow 10\cos 60 i + 10\sin 60 j$

$\tilde{b} \Rightarrow -6\cos 45 i + 6\sin 45 j$

$\tilde{c} \Rightarrow -7\cos 0 i + 7\sin 0 j$

$-6.24 i + 12.90 j$



$|\tilde{r}| = \sqrt{(-6.24)^2 + (12.9)^2}$

$= 14.33\text{N}$

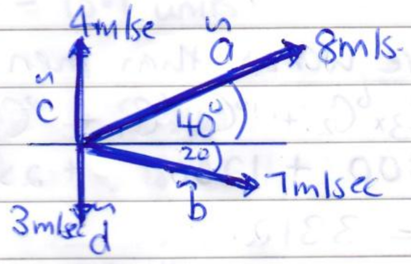
$\tan\theta = \dfrac{12.9}{6.24}$

$\theta = 64.2°$

$\therefore$ bearing $\Rightarrow 270 + 64.2$

$= 334°$

2.



$\tilde{a} \Rightarrow 8\cos 40 i + 8\sin 40 j$

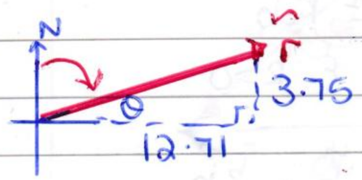
$\tilde{b} \Rightarrow 7\cos 20 i - 7\sin 20 j$

$\tilde{c} \Rightarrow 4\cos 90 i + 4\sin 90 j$

$\tilde{d} \Rightarrow 3\cos 90 i - 3\sin 90 j$

$12.71 i + 3.75 j$

2



$|\tilde{r}| = \sqrt{(12.71)^2 + (3.75)^2}$

$= 13.2$ m/sec

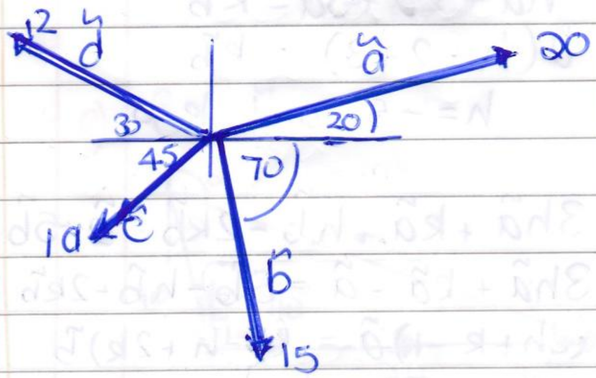
$\theta = \tan^{-1}\left(\dfrac{3.75}{12.71}\right)$

$\theta = 16.4°$

$\therefore$ bearing $= 90 - 16.4$

$= 074°$

3.



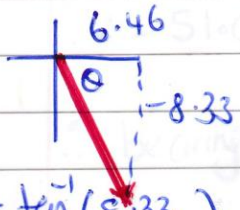
$\tilde{a} \Rightarrow 20\cos 20 i + 20\sin 20 j$

$\tilde{b} \Rightarrow 15\cos 70 i - 15\sin 70 j$

$\tilde{c} \Rightarrow -10\cos 45 i - 10\sin 45 j$

$\tilde{d} \Rightarrow -12\cos 30 i + 12\sin 30 j$

$6.46 i - 8.33 j$



$|\tilde{r}| = \sqrt{6.46^2 + 8.33^2}$

$|\tilde{r}| = 10.54$ units

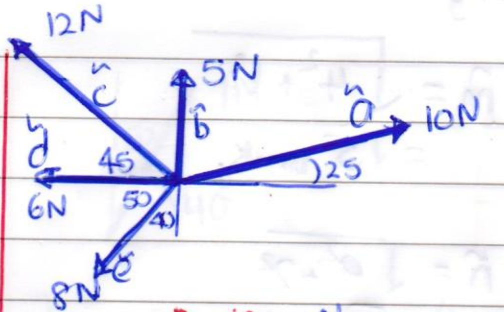
$\theta = \tan^{-1}\left(\dfrac{8.33}{6.46}\right)$

$\theta = 52.2$

bearing $= 90 + 52.2$

$= 142°$

4.



$\tilde{a} \Rightarrow 10\cos 25 i + 10\sin 25 j$

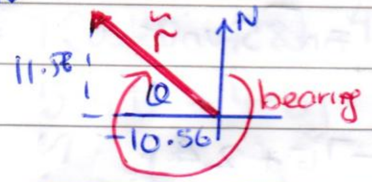
$\tilde{b} = 5\cos 90 i + 5\sin 90 j$

$\tilde{c} = -12\cos 45 i + 12\sin 45 j$

$\tilde{d} = -6\cos 0 i + 6\sin 0 j$

$\tilde{e} = -8\cos 50 i - 8\sin 50 j$

$-10.56 i + 11.58 j$



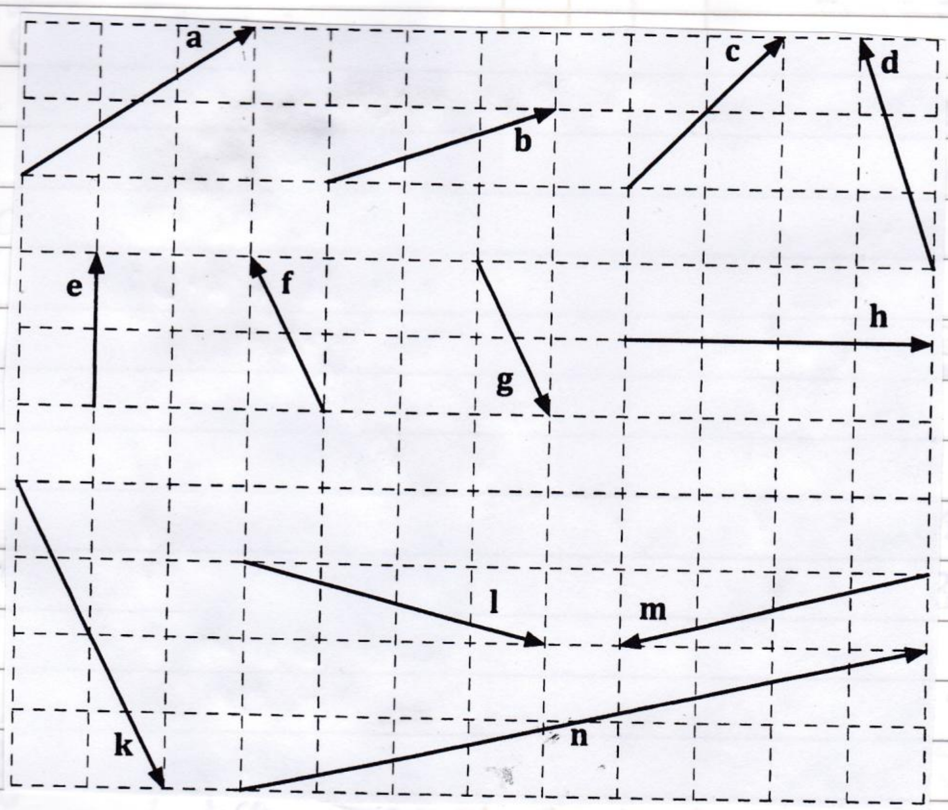
$|\tilde{r}| = \sqrt{(10.56)^2 + (11.58)^2}$

$|\tilde{r}| = 15.7N$

$\theta = \tan^{-1}\left(\frac{11.58}{10.56}\right)$ $\theta = 47.6^\circ$

$\therefore$ bearing $\Rightarrow 270 + 47.6^\circ = 318^\circ$

5.



a) $\tilde{a} = 3i + 2j$

b) $\tilde{b} = 3i + j$

c) $\tilde{c} = 2i + 2j$

d) $\tilde{d} = -i + 3j$

e) $\tilde{e} = 0i + 2j$

f) $\tilde{f} = -i + 2j$

g) $\tilde{g} = i - 2j$

h) $\tilde{h} = 4i + 0j$

k) $\tilde{k} = 2i - 4j$

l) $\tilde{l} = 4i - j$

m) $\tilde{m} = -4i - j$

n) $\tilde{n} = 9i + 2j$

$= \sqrt{[*]^2 + [*]^2}$ * dont need to worry about -ve.

6 a) $|\tilde{a}| = \sqrt{3^2+2^2}$
$= \sqrt{13}$ units

b) $|\tilde{b}| = \sqrt{3^2+1^2}$
$= \sqrt{10}$ units

c) $|\tilde{c}| = \sqrt{2^2+2^2}$
$= \sqrt{8}$
$= 2\sqrt{2}$ units

d) $|\tilde{d}| = \sqrt{1^2+3^2}$
$= \sqrt{10}$ units

e) $|\tilde{e}| = \sqrt{0^2+2^2}$
$= 2$ units

f) $|\tilde{f}| = \sqrt{1^2+2^2}$
$= \sqrt{5}$ units

g) $|\tilde{g}| = \sqrt{1^2+2^2}$
$= \sqrt{5}$ units

h) $|\tilde{h}| = \sqrt{4^2+0^2}$
$= 4$ units

k) $|\tilde{k}| = \sqrt{2^2+4^2}$
$= \sqrt{20}$
$= 2\sqrt{5}$ units

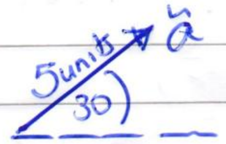
l) $|\tilde{l}| = \sqrt{4^2+1^2}$
$= \sqrt{5}$ units

m) $\tilde{m} = \sqrt{4^2+1^2}$
$= \sqrt{5}$ units.

n) $\tilde{n} = \sqrt{9^2+2^2}$
$= \sqrt{85}$ units

7. $-7i + 24j$ N
$|\tilde{a}| = \sqrt{7^2+24^2}$
$= \sqrt{625}$
$= 25$ N

8a)

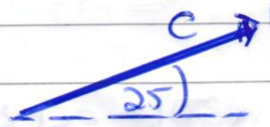


$\tilde{a} = 5\cos 30 i + 5\sin 30 j$
$= 4.3i + 2.5j$

b)

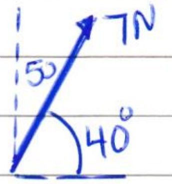
$\tilde{b} = 7\cos 60 i + 7\sin 60 j$
$= 3.5i + 6.1j$

c)



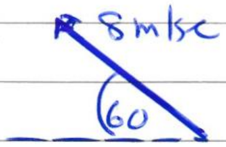
$\tilde{c} = 10\cos 25 i + 10\sin 25 j$
$= 9.1i + 4.2j$

d).



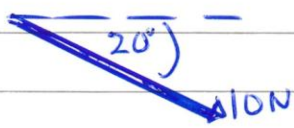
$\hat{d} = 7\cos 40 i + 7\sin 40 j$
$= 5{\cdot}4 i + 4{\cdot}5 j$

e)



$\hat{e} = -8\cos 60 i + 8\sin 60 j$
$= -4 i + 6{\cdot}9 j$

f)

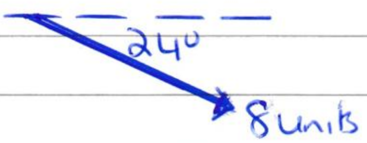


$\hat{f} = 10\cos 20 i - 10\sin 20 j$
$= 9{\cdot}4 i - 3{\cdot}4 j$

g)

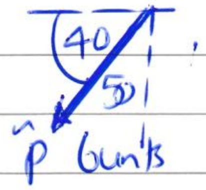
$\hat{g} = -4\cos 50 i + 4\sin 50 j$
$= -2{\cdot}6 i + 3{\cdot}1 j$

h)



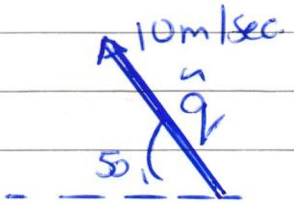
$\hat{h} = 8\cos 24 i - 8\sin 24 j$
$= 7{\cdot}3 i - 3{\cdot}4 j$

i)



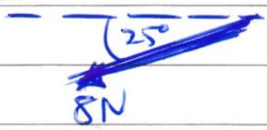
$\hat{p} = -6\cos 40 i - 6\sin 40 j$
$= -4{\cdot}6 i - 3{\cdot}9 j$

j)



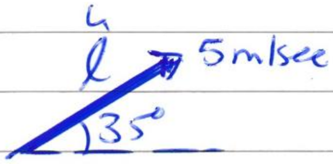
$\hat{q} = -10\cos 50 i + 10\sin 50 j$
$= -6{\cdot}4 i + 7{\cdot}7 j$

k)



$\hat{k} = -8\cos 25 i - 8\sin 25 j$
$= -7{\cdot}3 i - 3{\cdot}4 j$

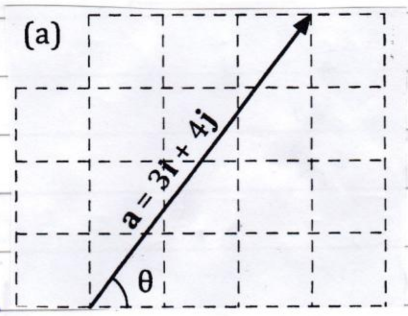
l)



$\hat{l} = 5\cos 35 i + 5\sin 35 j$
$= 4{\cdot}1 i + 2{\cdot}9 j$

9.

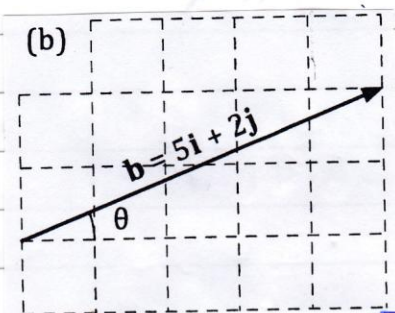
a)



$|\vec{a}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$ units

$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.1^\circ$

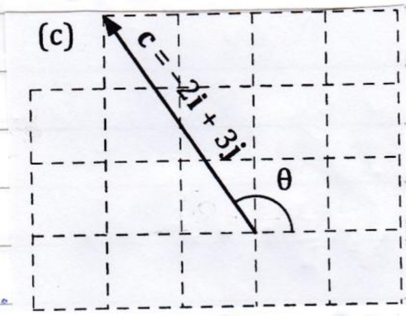
b)



$|\vec{b}| = \sqrt{5^2 + 2^2} = \sqrt{29}$ units

$\theta = \tan^{-1}\left(\frac{2}{5}\right) \quad \theta = 21.8^\circ$

c)

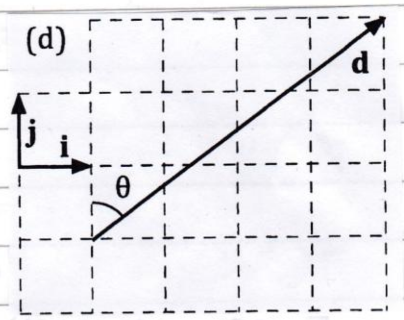


$|\vec{c}| = \sqrt{2^2 + 3^2} = \sqrt{13}$ units

$\alpha = \tan^{-1}\left(\frac{3}{2}\right) = 56.3$

$\therefore \theta = 180 - 56.3$
$= 123.7^\circ$

d)



$\vec{d} = 4i + 3j$

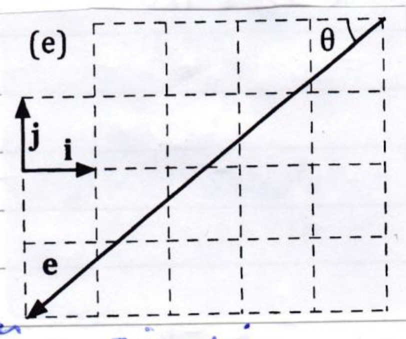
$|\vec{d}| = \sqrt{4^2 + 3^2}$

$= 5$ units

$\theta = \tan^{-1}\left(\frac{3}{4}\right)$

$\theta = 53.1^\circ$

e)



$\vec{e} = -5i - 4j$

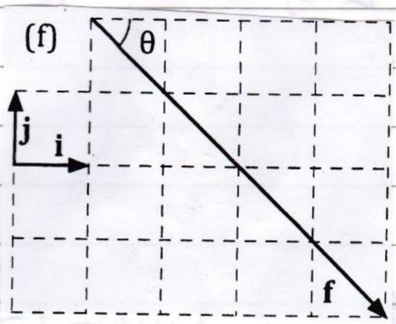
$|\vec{e}| = \sqrt{5^2 + 4^2}$

$= \sqrt{41}$ units

$\theta = \tan^{-1}\left(\frac{4}{5}\right)$

$\theta = 38.7^\circ$

f).



$\hat{f} = 4i - 4j$

$|\hat{f}| = \sqrt{4^2 + 4^2}$

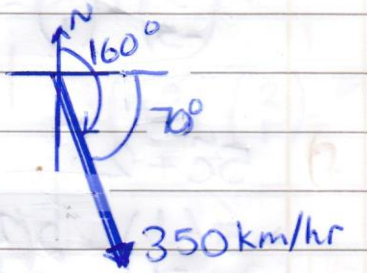
$= \sqrt{32}$

$= 4\sqrt{2}$ units

$\theta = \tan^{-1}\left(\frac{4}{4}\right)$

$\theta = 45^\circ$

10.



a) $-350\sin 70 = 328.89$

≈ 330 km/hr

b) $+350\cos 70 = 119.7$ km/hr

≈ 120 km/hr

11.

$-5i + 8j$

$|\hat{r}| = \sqrt{5^2 + 8^2} = \sqrt{89}$ units

$\theta = \tan^{-1}\left(\frac{8}{5}\right)$ $\theta = 57.99^\circ$

bearing $= 270 + 58 = 328^\circ$

12 $\hat{a} = 2i + 3j$

$\hat{b} = i + 4j$

a) $\hat{a} + \hat{b} = \begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}1\\4\end{pmatrix}$

$= 3i + 7j$

b) $\hat{a} - \hat{b} = \begin{pmatrix}2\\3\end{pmatrix} - \begin{pmatrix}1\\4\end{pmatrix}$

$= i - j$

c) $\hat{b} - \hat{a} = \begin{pmatrix}1\\4\end{pmatrix} - \begin{pmatrix}2\\3\end{pmatrix}$

$= -i + j$

d) $2\hat{a} = 2 \times \begin{pmatrix}2\\3\end{pmatrix}$

$= 4i + 6j$

e) $3\hat{b} = 3 \times \begin{pmatrix}1\\4\end{pmatrix}$

$= 3i + 12j$

f) $2\hat{a} + 3\hat{b}$

$2\begin{pmatrix}2\\3\end{pmatrix} + 3\begin{pmatrix}1\\4\end{pmatrix}$

$= \begin{pmatrix}4\\6\end{pmatrix} + \begin{pmatrix}3\\12\end{pmatrix}$

$= 7i + 18j$

g. $2\hat{a} - 3\hat{b} = 2\begin{pmatrix}2\\3\end{pmatrix} - 3\begin{pmatrix}1\\4\end{pmatrix}$

$= \begin{pmatrix}4\\6\end{pmatrix} - \begin{pmatrix}3\\12\end{pmatrix} = i - 6j$

h) $-2\hat{a} + 3\hat{b} = -2\begin{pmatrix}2\\3\end{pmatrix} + 3\begin{pmatrix}1\\4\end{pmatrix}$

$= \begin{pmatrix}-4\\-6\end{pmatrix} + \begin{pmatrix}3\\12\end{pmatrix} = -i + 6j$

i) $|\hat{a}| = \sqrt{2^2 + 3^2}$
$= \sqrt{13}$ units

j) $|\hat{b}| = \sqrt{1^2 + 4^2}$
$= \sqrt{17}$ units

k) $|\hat{a}| + |\hat{b}|$
$= \sqrt{13} + \sqrt{17}$ units

l) $|\hat{a} + \hat{b}| = \begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}1\\4\end{pmatrix} = \begin{pmatrix}3\\7\end{pmatrix}$

$= \sqrt{3^2 + 7^2}$
$= \sqrt{58}$ units

13. $\hat{c} = i - j \quad \hat{d} = 2i + j$

a) $2\hat{c} + \hat{d} = 2\begin{pmatrix}1\\-1\end{pmatrix} + \begin{pmatrix}2\\1\end{pmatrix}$

$= \begin{pmatrix}2\\-2\end{pmatrix} + \begin{pmatrix}2\\1\end{pmatrix}$

$= 4i - j$

b) $\hat{c} - \hat{d} = \begin{pmatrix}1\\-1\end{pmatrix} - \begin{pmatrix}2\\1\end{pmatrix}$

$= -i - 2j$

c) $\hat{d} - \hat{c} = \begin{pmatrix}2\\1\end{pmatrix} - \begin{pmatrix}1\\-1\end{pmatrix}$

$= i + 2j$

d) $5\hat{c} = 5\begin{pmatrix}1\\-1\end{pmatrix}$

$= 5i - 5j$

e) $5\hat{c} + \hat{d} = 5\begin{pmatrix}1\\-1\end{pmatrix} + \begin{pmatrix}2\\1\end{pmatrix}$

$= \begin{pmatrix}5\\-5\end{pmatrix} + \begin{pmatrix}2\\1\end{pmatrix} = 7i - 4j$

f) $5\hat{c} + 2\hat{d}$

$5\begin{pmatrix}1\\-1\end{pmatrix} + 2\begin{pmatrix}2\\1\end{pmatrix}$

$= \begin{pmatrix}5\\-5\end{pmatrix} + \begin{pmatrix}4\\2\end{pmatrix} = 9i - 3j$

g) $2\hat{c} + 5\hat{d}$

$2\begin{pmatrix}1\\-1\end{pmatrix} + 5\begin{pmatrix}2\\1\end{pmatrix}$

$= \begin{pmatrix}2\\-2\end{pmatrix} + \begin{pmatrix}10\\5\end{pmatrix}$

$= 12i + 3j$

h) $2\hat{c} - \hat{d}$

$2\begin{pmatrix}1\\-1\end{pmatrix} - \begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}2\\-2\end{pmatrix} - \begin{pmatrix}2\\1\end{pmatrix}$

$= 0i - 3j$

i) $|\hat{d} - 2\hat{c}|$ $\quad \begin{pmatrix}2\\1\end{pmatrix} - 2\begin{pmatrix}1\\-1\end{pmatrix}$

$= \begin{pmatrix}2\\1\end{pmatrix} - \begin{pmatrix}2\\-2\end{pmatrix} = 0i + 3j$

$\sqrt{0^2+3^2} = 3$ units

j) $|\hat{c}| + |\hat{d}|$

$= \sqrt{1^2+1^2} + \sqrt{2^2+1^2}$

$= \sqrt{2} + \sqrt{5}$ units

≈ 3.65 units

k) $|\hat{c} + \hat{d}| = \begin{pmatrix}1\\-1\end{pmatrix} + \begin{pmatrix}2\\1\end{pmatrix}$

$= 3i + 0j$

$= \sqrt{3^2+0^2}$

$= 3$ units

l) $|\hat{c} - \hat{d}| = \begin{pmatrix}1\\-1\end{pmatrix} - \begin{pmatrix}2\\1\end{pmatrix}$

$= -i - 2j$

$\sqrt{1^2+2^2}$

$= \sqrt{5}$ units

14 $\hat{a} = \langle 5,4 \rangle \quad \hat{b} = \langle 2,-3 \rangle$

a) $\hat{a} + \hat{b} = \begin{pmatrix}5\\4\end{pmatrix} + \begin{pmatrix}2\\-3\end{pmatrix}$

$= \langle 7, 1 \rangle$

b) $\hat{a} - \hat{b} = \begin{pmatrix}5\\4\end{pmatrix} - \begin{pmatrix}2\\-3\end{pmatrix}$

$= \langle 3, 7 \rangle$

c) $2\hat{a} = 2 \times \langle 5,4 \rangle$

$= \langle 10, 8 \rangle$

d) $3\hat{a} + \hat{b} = 3\begin{pmatrix}5\\4\end{pmatrix} + \begin{pmatrix}2\\-3\end{pmatrix}$

$= \begin{pmatrix}15\\12\end{pmatrix} + \begin{pmatrix}2\\-3\end{pmatrix}$

$= \langle 17, 9 \rangle$

e) $2\hat{b} - \hat{a} = 2\begin{pmatrix}2\\-3\end{pmatrix} - \begin{pmatrix}5\\4\end{pmatrix}$

$= \begin{pmatrix}4\\-6\end{pmatrix} - \begin{pmatrix}5\\4\end{pmatrix}$

$= \langle -1, -10 \rangle$

f) $|\hat{a}| = \sqrt{5^2+4^2}$

$= \sqrt{41}$ units

g) $|\hat{a} + \hat{b}| = \begin{pmatrix}5\\4\end{pmatrix} + \begin{pmatrix}2\\-3\end{pmatrix}$

$= \begin{pmatrix}7\\1\end{pmatrix} = \sqrt{7^2+1^2} = \sqrt{50}$

$= 5\sqrt{2}$ units

h) $|\hat{a}| + |\hat{b}|$
$= \sqrt{5^2+4^2} + \sqrt{2^2+3^2}$
$= \sqrt{41} + \sqrt{13}$ units
≈ 10.01 units

15 $\hat{c} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $\hat{d} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

a) $\hat{c} + \hat{d} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

b) $\hat{c} - \hat{d} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

c) $\hat{d} - \hat{c} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \end{pmatrix}$

d) $2\hat{c} + \hat{d} = 2\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 8 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$

e) $\hat{c} + 2\hat{d} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + 2\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

f) $\hat{c} - 2\hat{d} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - 2\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

g) $|\hat{c} - 2\hat{d}|$ ∝ (h) same

$\sqrt{5^2+4^2} = \sqrt{41}$ units

h) $|2\hat{d} - \hat{c}| = 2\begin{pmatrix} -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} -5 \\ -4 \end{pmatrix}$

$= \sqrt{5^2+4^2}$
$= \sqrt{41}$ units

16. $\hat{a} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$ $\hat{b} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

a) $|\hat{a}| = \sqrt{2^2+7^2}$
$= \sqrt{53}$ units

b) $|\hat{b}| = \sqrt{2^2+3^2}$
$= \sqrt{13}$ units

c) $|\hat{a} + \hat{b}| = \begin{pmatrix} 2 \\ 7 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ 10 \end{pmatrix}$
$= \sqrt{0^2+10^2} = 10$ units

d) $|2\hat{a}| = 2\begin{pmatrix} 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 4 \\ 14 \end{pmatrix}$

$= \sqrt{4^2+14^2}$
$= \sqrt{212}$
$= 2\sqrt{53}$ units

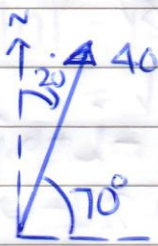
e) $|\hat{a}-\hat{b}| = \begin{pmatrix}2\\7\end{pmatrix} - \begin{pmatrix}-2\\3\end{pmatrix} = \begin{pmatrix}4\\4\end{pmatrix}$

$= \sqrt{4^2+4^2}$

$= \sqrt{32}$

$= 4\sqrt{2}$ units

17.



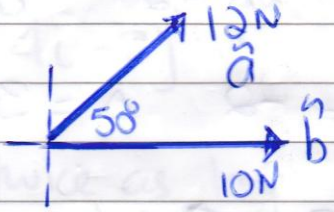
a) $4000\sin 70$

$= 3758 \approx 3760N$

b) $4000\cos 70$

$= 1368 \approx 1370N$

18.

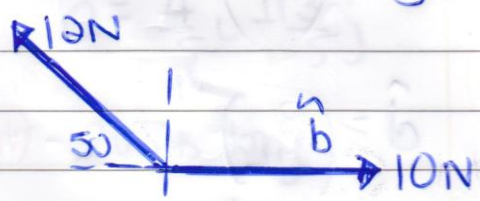


$\hat{a} \Rightarrow 12\cos 50i + 12\sin 50j$

$\hat{b} \Rightarrow 10\cos 0i + 10\sin 0j$

$17.7i + 9.2j$

19.

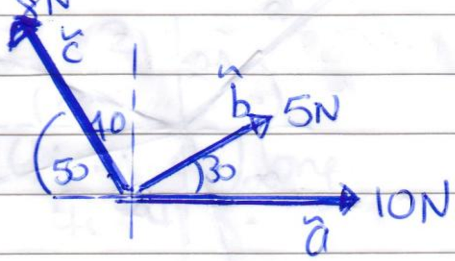


$\hat{a} \Rightarrow -12\cos 50i + 12\sin 50j$

$\hat{b} \Rightarrow 10\cos 0i + 10\sin 0j$

$2.3i + 9.2j$

20.



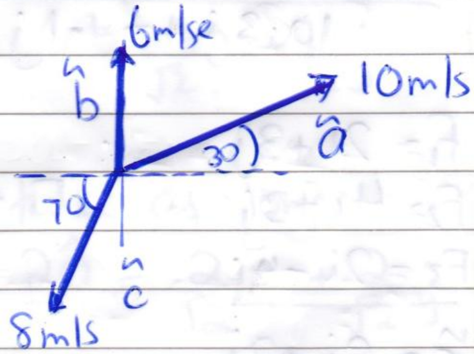
$\hat{a} \Rightarrow 10\cos 0i + 10\sin 0j$

$\hat{b} \Rightarrow 5\cos 30i + 5\sin 30j$

$\hat{c} \Rightarrow -8\cos 50i + 8\sin 50j$

$9.2i + 8.6j$

21.



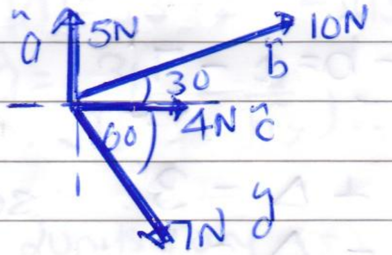
$\hat{a} \Rightarrow 10\cos 30i + 10\sin 30j$

$\hat{b} = 6\cos 90i + 6\sin 90j$

$\hat{c} = -8\cos 70i - 8\sin 70j$

$5.9i + 3.5j$

22.



$\hat{a} = 5\cos 90i + 5\sin 90j$

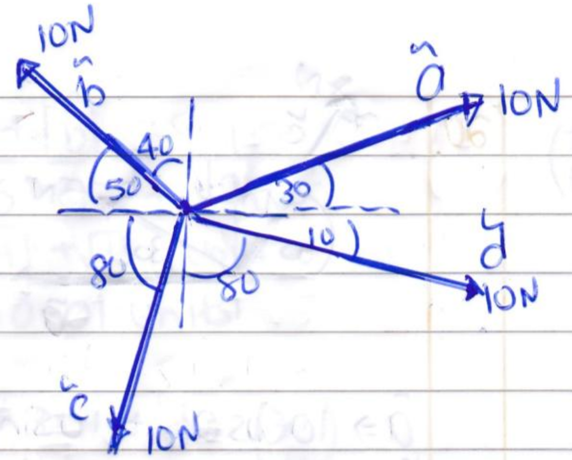
$\hat{b} = 10\cos 30i + 10\sin 30j$

$\hat{c} = 4\cos 0i + 4\sin 0j$

$\hat{d} = 7\cos 60i - 7\sin 60j$

$16.2i + 3.9j$

23.



$\hat{a} = 10\cos 30 i + 10\sin 30 j$
$\hat{b} = -10\cos 50 i + 10\sin 50 j$
$\hat{c} = -10\cos 80 i - 10\sin 80 j$
$\hat{d} = 10\cos 10 i - 10\cos 10 j$
$10.3 i + 1.1 j$

24. $F_1 = 2i + 3j$
$F_2 = 4i + 3j$ $\quad F_1 + F_2 + F_3$
$F_3 = 2i - 4j$

$\hat{r} = 8i + 2j$

$|\hat{r}| = \sqrt{8^2 + 2^2} = \sqrt{68}$
$= 2\sqrt{17}$ N

25. $\hat{a} + \hat{b} = 3i + j \quad \hat{a} = \begin{pmatrix} * \\ \square \end{pmatrix}$

$\hat{a} - \hat{b} = i - 7j \quad \hat{b} = \begin{pmatrix} \Delta \\ ✡ \end{pmatrix}$

$* + \Delta = 3$ solve on
$* - \Delta = 1$ calc

$* = 2 \quad \Delta = 1$

$\square + ✡ = 1$ solve
$\square - ✡ = -7$ on calc

$\square = -3 \quad ✡ = 4$

$\therefore \hat{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \quad \hat{b} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

26. $2\hat{c} + \hat{d} = -i + 6j$
$2\hat{c} + 2\hat{d} = 2i - 10j$

$\hat{c} = \begin{pmatrix} * \\ \square \end{pmatrix} \quad \hat{d} = \begin{pmatrix} \Delta \\ ✡ \end{pmatrix}$

$2* + \Delta = -1$ solve on
$2* + 2\Delta = 2$ calc

$* = -2 \quad \Delta = 3$

$2\square + ✡ = 6$ solve
$2\square + 2✡ = -10$ on calc
$\square = 11 \quad ✡ = -16$

$\therefore \hat{c} = \begin{pmatrix} -2 \\ 11 \end{pmatrix}$

$\hat{d} = \begin{pmatrix} 3 \\ -16 \end{pmatrix}$

Ex 4B.

1\. a). i) $4i + 3j$

ii) twice as long
$= 8i + 6j$

iii) $|\hat{a}| = \sqrt{4^2+3^2}$
$= \sqrt{25} = 5$

$\hat{a} = \frac{4}{5}i + \frac{3}{5}j$

iv) 2 units long
ie $2 \times \hat{a}$
$= \frac{8}{5}i + \frac{6}{5}j$

b) i) $4i - 3j$

ii) twice as long
$8i - 6j$

iii) $|\hat{a}| = \sqrt{4^2+3^2}$
$= 5$ units

$\hat{a} = \frac{4}{5}i - \frac{3}{5}j$

iv) 2 units long

$\therefore 2 \times \hat{a}$

$= \frac{8}{5}i - \frac{6}{5}j$

c i) $2i + 2j$

ii) twice as long
$4i + 4j$

iii) $|\hat{c}| = \sqrt{2^2+2^2}$
$= \sqrt{8}$
$= 2\sqrt{2}$

$\therefore \hat{c} = \frac{2}{2\sqrt{2}}i + \frac{2}{2\sqrt{2}}j$

$= \frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j$

iv) twice as long
$2\hat{a} = \frac{2}{\sqrt{2}}i + \frac{2}{\sqrt{2}}j$
$= \sqrt{2}i + \sqrt{2}j$

d) i) $3i - 2j$

ii) twice as long $\Rightarrow 6i - 4j$

iii) $|\hat{d}| = \sqrt{3^2+2^2} = \sqrt{13}$
$\hat{a} = \frac{3}{\sqrt{13}}i - \frac{2}{\sqrt{13}}j$

iv) 2 units long
$2\hat{a}$
$= \frac{6}{\sqrt{13}}i - \frac{4}{\sqrt{13}}j$

2. $\hat{a} = \begin{pmatrix}-3\\4\end{pmatrix}$ $\hat{b} = \begin{pmatrix}2\\1\end{pmatrix}$ $\hat{c} = \begin{pmatrix}3\\-2\end{pmatrix}$

a) $|\hat{b}| = \sqrt{2^2+1^2} = \sqrt{5}$

$\hat{b} = \frac{2}{\sqrt{5}}i + \frac{1}{\sqrt{5}}j$

b) $|\hat{a}| = \sqrt{3^2+4^2} = 5$

size of $\hat{a}$ but in direction of $\hat{b}$

$= 5\left(\frac{2}{\sqrt{5}}i + \frac{1}{\sqrt{5}}j\right)$

$= \frac{10}{\sqrt{5}}i + \frac{5}{\sqrt{5}}j$

c) $|\hat{c}| = \sqrt{3^2+2^2} = \sqrt{13}$

$\hat{a} = \frac{3}{5}i + \frac{4}{5}j$

∴ size of $\hat{c}$ but in direction of $\hat{a}$

$\sqrt{13}\left(\frac{3}{5}i + \frac{4}{5}j\right)$

d) $\hat{a} + \hat{b} + \hat{c} = \begin{pmatrix}-3\\4\end{pmatrix} + \begin{pmatrix}2\\1\end{pmatrix} + \begin{pmatrix}3\\-2\end{pmatrix}$

$= \begin{pmatrix}2\\3\end{pmatrix}$

$|\hat{r}| = \sqrt{2^2+3^2} = \sqrt{13}$

size of $\hat{a}$ $5\left(\frac{2}{\sqrt{13}}i + \frac{3}{\sqrt{13}}j\right)$

3. $\hat{a} = \begin{pmatrix}2\\-4\end{pmatrix}$ $\hat{b} = \begin{pmatrix}4\\2\end{pmatrix}$

$\hat{c} = \begin{pmatrix}1\\-8\end{pmatrix}$ $\hat{d} = \begin{pmatrix}1\\-2\end{pmatrix}$

$\hat{e} = \begin{pmatrix}4\\-2\end{pmatrix}$

a) parallel ie same direction

$\hat{a}$ & $\hat{d}$

$\begin{pmatrix}2\\-4\end{pmatrix} \Rightarrow 2\begin{pmatrix}1\\-2\end{pmatrix} \Rightarrow 2\hat{d}$

b) $\hat{a} + \hat{b} + \hat{c} + \hat{d} + \hat{e} = \hat{r}$

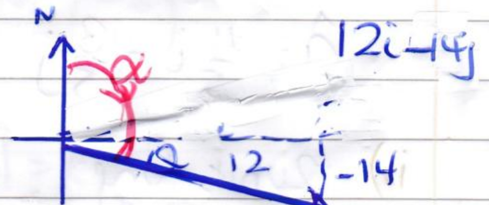
$\begin{pmatrix}2\\-4\end{pmatrix} + \begin{pmatrix}4\\2\end{pmatrix} + \begin{pmatrix}1\\-8\end{pmatrix} + \begin{pmatrix}1\\-2\end{pmatrix} + \begin{pmatrix}4\\-2\end{pmatrix}$

$= \begin{pmatrix}12\\-14\end{pmatrix}$

c) $|\hat{r}| = \sqrt{12^2+4^2}$

$= \sqrt{340}$

$= 2\sqrt{85}$



$\theta = \tan^{-1}\left(\frac{14}{12}\right)$

$\theta = 49.4°$

∴ bearing

$\alpha = 90 + 49.4 = 139°$

4. $\hat{a} = \begin{pmatrix} w \\ 3 \end{pmatrix}$ $\hat{b} = \begin{pmatrix} -1 \\ x \end{pmatrix}$

$\hat{c} = \begin{pmatrix} 0.5 \\ y \end{pmatrix}$ $\hat{d} = \begin{pmatrix} -1 \\ -z \end{pmatrix}$

$|\hat{a}| = 5$
ie $\sqrt{w^2 + 3^2} = 5$ solve on calc.

$w = -4$

$\therefore \hat{a} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$

$\hat{b}$ is parallel to $\hat{a}$

ie $\begin{pmatrix} -4 \\ 3 \end{pmatrix} = \lambda \begin{pmatrix} -1 \\ x \end{pmatrix}$

so $-4 = -\lambda$
$\lambda = 4$

$\therefore 3 = 4(x)$
$\frac{3}{4} = x$

$\hat{c}$ ie $|\hat{c}| = 1$
$1 = \sqrt{0.5^2 + y^2}$ solve

$y = \pm \frac{\sqrt{3}}{2}$

$|\hat{a} + \hat{d}| = 13$ $*w = -4$

$\left| \begin{pmatrix} -4 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ -z \end{pmatrix} \right| = 13$

$\left| \begin{matrix} -5 \\ 3 - z \end{matrix} \right| = 13$

$5^2 + (3 - z)^2 = 13^2$ solve
$z = -9$ or 15

5. $\hat{p} = 0.6i - aj$ $\hat{q} = bi + cj$

$\hat{r} = di + ej$ $\hat{s} = fi + gj$

$\hat{p}$ ie $(0.6)^2 + (a)^2 = 1$
$\therefore a = 0.8$ & +ve

$\therefore \hat{p} = 0.6i - 0.8j$

$\hat{q} = 5$ units & same direction as $\hat{p}$
ie $\hat{q} = 5(0.6i + 0.8j)$
$= 3i - 4j$
ie $b = 3$ $c = -4$

$\hat{r} + 2\hat{q} = \begin{pmatrix} 11 \\ -20 \end{pmatrix}$
$\begin{pmatrix} d \\ e \end{pmatrix} + \begin{pmatrix} 6 \\ -8 \end{pmatrix} = \begin{pmatrix} 11 \\ -20 \end{pmatrix}$

$\therefore d = 5$
$e = -12$

$\hat{s} = k\hat{r}$ but $|\hat{s}| = 5$

$\uparrow$ magnitude of $\hat{q}$

$\hat{s} = k\begin{pmatrix} d \\ e \end{pmatrix} = k\begin{pmatrix} 5 \\ -12 \end{pmatrix}$

$|(5k)^2 + (12k)^2| = 5^2$

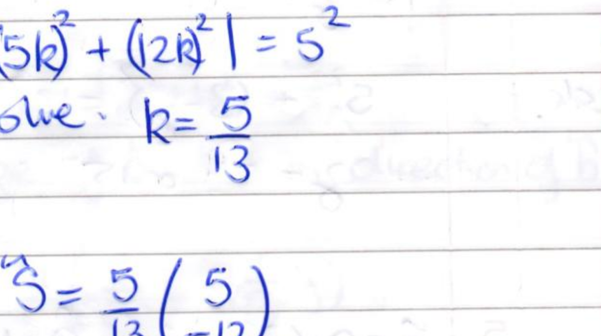
solve. $k = \frac{5}{13}$

$\therefore \hat{s} = \frac{5}{13}\begin{pmatrix} 5 \\ -12 \end{pmatrix}$

$= \frac{25}{13}i - \frac{60}{13}j$

$\therefore f = \frac{25}{13} \quad g = -\frac{60}{13}$

6.



$\hat{a} \Rightarrow 7\cos 30 i + 7\sin 30 j$
$\hat{b} \Rightarrow 6\cos 90 i + 6\sin 90 j$
$\hat{c} \Rightarrow 10\cos 45 i + 10\sin 45 j$
$\hat{d} \Rightarrow -4\cos 35 i + 4\sin 35 j$
$\quad\quad 9.9i + 18.9j$

$\hat{a} + \hat{b} + \hat{c} + \hat{d} + \hat{e} = 0 \quad \therefore$

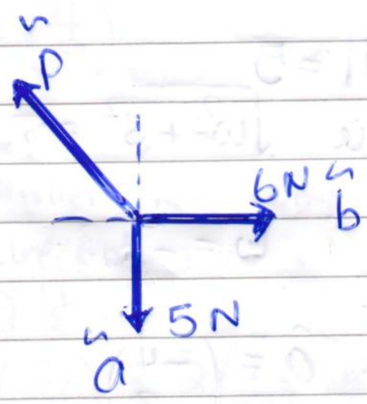
$\hat{e} = -9.9i - 18.9j$

$|\hat{r}| = |\hat{a} + \hat{b} + \hat{c} + \hat{d}|$

$= \sqrt{9.9^2 + 18.9^2}$

$= 21.3$ units

7.

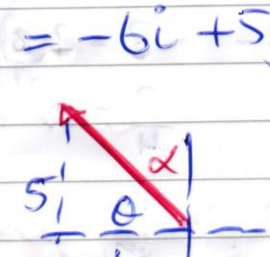


$\hat{a} + \hat{b} + \hat{p} = 0$

$\therefore \hat{p} = -(\hat{a} + \hat{b})$

$\hat{a} \Rightarrow 6i + 0j$
$\hat{b} \Rightarrow 0i - 5j$
$\quad 6i - 5j$

$\therefore \hat{p} = -6i + 5j$



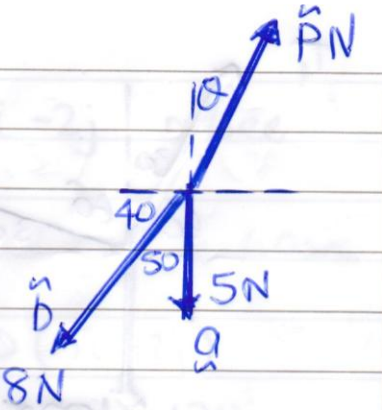
$\theta = \tan^{-1}\left(\frac{5}{6}\right)$

$\theta = 39.8^\circ \quad \alpha = 50^\circ$

$|6^2 + 5^2| = \sqrt{61}$

$\doteq 7.8$ N

8.



$\hat{a} + \hat{b} + \hat{p} = 0 \quad \therefore \hat{p} = -(\hat{a} + \hat{b})$

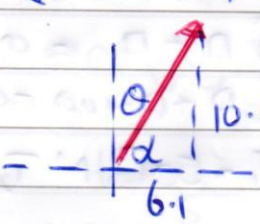
$\hat{a} \Rightarrow 0i - 5j$

$\hat{b} \Rightarrow -8\cos 40 i - 8\sin 40 j$

$-6.128i - 10.142j$

$\therefore \hat{p} = 6.1i + 10.1j$

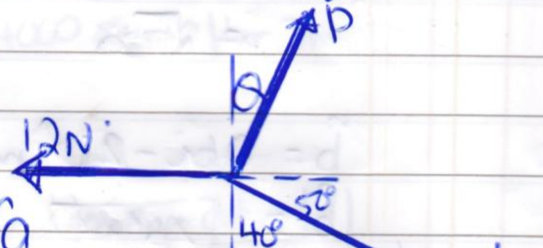
$|\hat{p}| = \sqrt{6.1^2 + 10.1^2} = 11.9\text{N}$



$\alpha = \tan^{-1}\left(\frac{10.1}{6.1}\right) \quad \alpha = 58.85$

$\therefore \theta = 31°$

9.



$\hat{a} + \hat{b} + \hat{p} = 0$

$\hat{a} \Rightarrow -12i + 0j$

$\hat{b} \Rightarrow 10\cos 50 i - 10\sin 50 j$

$-5.6i - 7.7j$

$\therefore \hat{p} = 5.6i + 7.7j$

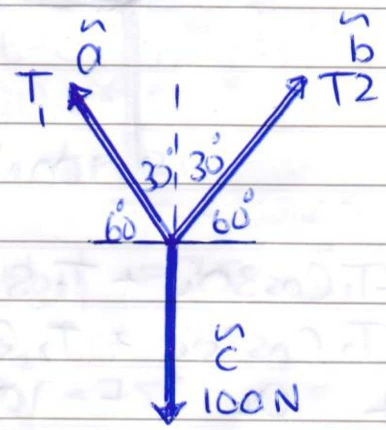
$|\hat{p}| = \sqrt{5.6^2 + 7.7^2}$

$= 9.5\text{N}$

$\alpha = \tan^{-1}\left(\frac{7.6}{5.6}\right)$

$\alpha = 54° \quad \therefore \theta = 36°$

10.



$\hat{a} + \hat{b} + \hat{c} = 0$

$\hat{a} \Rightarrow -T_1\cos 60 i + T_1 \sin 60 j$

$\hat{b} \Rightarrow T_2 \cos 60 i + T_2 \sin 60 j$

$\hat{c} \Rightarrow 0i - 100j$

$0i + 0j$

need to solve simultaneous equations

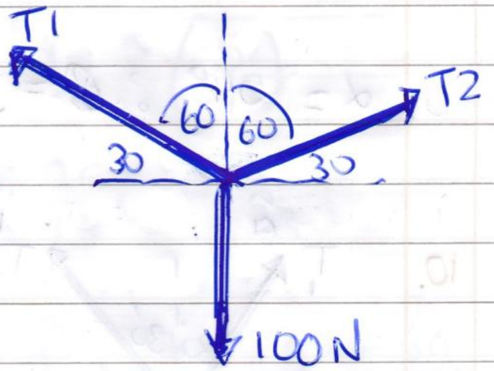
"i" $-T_1 \cos 60 + T_2 \cos 60 = 0$

"j" $T_1 \sin 60 + T_2 \sin 60 - 100 = 0$

$$\begin{cases} \square + \square = 0 \\ \square + \square + \square = 0 \end{cases}$$

$$T_1 = T_2 = \frac{100\sqrt{3}}{3} \overset{*}{=} \frac{100}{\sqrt{3}}$$

11.



$$\begin{array}{rl} & -T_1 \cos 30 i + T_1 \sin 30 j \\ & T_2 \cos 30 i + T_2 \sin 30 j \\ + & 0i - 100j \\ \hline & 0i + 0j \end{array}$$

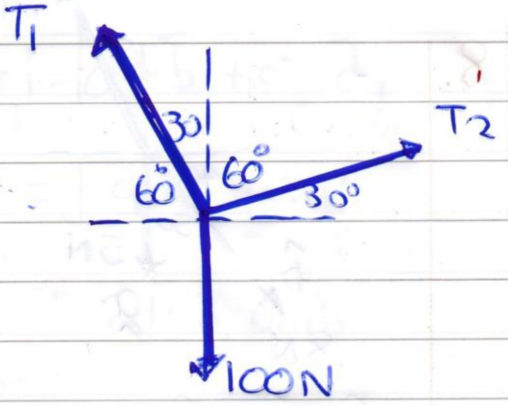
"i" $-T_1 \cos 30 + T_2 \cos 30$

"j" $T_1 \sin 30 + T_2 \sin 30 - 100$

$$\begin{cases} \square + \square = 0 \\ \square + \square - 100 = 0 \end{cases}$$

$$T_1 = T_2 = 100N$$

12.



$$\begin{array}{rl} & -T_1 \cos 60 i + T_1 \sin 60 j \\ & T_2 \cos 30 i + T_2 \sin 30 j \\ + & 0i - 100j \\ \hline & 0i + 0j \end{array}$$

"i" $-T_1 \cos 60 + T_2 \cos 30$

"j" $T_1 \sin 60 + T_2 \sin 30 - 100$

$$\begin{cases} \square + \square = 0 \\ \square + \square - 100 = 0 \end{cases}$$

$T_1 = 50\sqrt{3}$ N $T_2 = 50$ N

13. $\hat{a} = 21i + 17j$ m/sec

$$|\hat{a}| = \sqrt{21^2 + 17^2} = \sqrt{730}$$

$\hat{b} = 26i - 2j$ m/sec

$$|\hat{b}| = \sqrt{26^2 + 2^2} = \sqrt{680}$$

$\hat{a}$ is moving faster

14. $5i - 2j$ m/sec

in one min ie 60sec

$60(5i - 2j)$

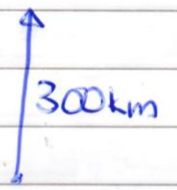
$= 300i - 120j$

$\sqrt{300^2 + 120^2} = 60\sqrt{29}$

$= 323.1$

$\approx 323m$

15.



no wind ⇒ can fly 75 m/sec

$s = \frac{d}{t}$ $\quad t = \frac{d}{s}$

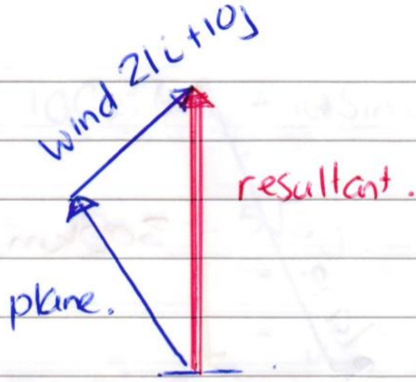
$t = \frac{300 \times 1000}{75}$

$t = 4000$ sec $\approx$ 1hr 7min

b) wind blowing at

$21i + 10j$

* need to adjust flight path.



* $s = \frac{d}{t}$ $\quad d = s \times t$

ie t(plane + wind) = 300km.

$t\begin{pmatrix} a \\ b \end{pmatrix} + t\begin{pmatrix} 21 \\ 10 \end{pmatrix} = \begin{pmatrix} 0 \\ 300000m \end{pmatrix}$

$at + 21t = 0$

$t(a + 21) = 0$

ie $\cancel{t=0}$ $\quad t = -21$

also $|ai + bj| = 75$

$a^2 + b^2 = 75^2$

$(21)^2 + b^2 = 75^2$ solve.

$b = 72$

∴ plane $= -21i + 72j$

∴ resultent ⇒ $\begin{pmatrix} -21 \\ 72 \end{pmatrix} + \begin{pmatrix} 21 \\ 10 \end{pmatrix} = \begin{pmatrix} 0 \\ 82 \end{pmatrix}$

$\sqrt{0^2 + 82^2} = 82$ m/sec

$t = \frac{300000}{82} = 3658.4$ sec

$\approx$ 1hr 1min

16.

$t(\text{plane} + \text{wind}) = \begin{pmatrix} 0 \\ -300000\,\text{m} \end{pmatrix}$

$t\begin{pmatrix} a \\ b \end{pmatrix} + t\begin{pmatrix} 21 \\ 10 \end{pmatrix} = \begin{pmatrix} 0 \\ -300000 \end{pmatrix}$

$at + 21t = 0$

$t(a+21) = 0$

$\therefore a = -21$

$|ai + bj| = 75$

$\sqrt{(-21)^2 + b^2} = 75$ * must be -ve solution

$\therefore b = -72$

$\therefore \text{plane} = \begin{pmatrix} -21 \\ -72 \end{pmatrix}$

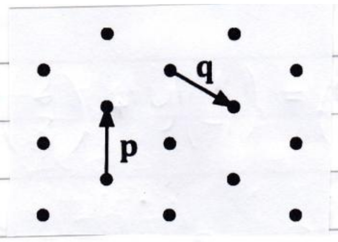
$\therefore \text{resultant} = \begin{pmatrix} -21 \\ -72 \end{pmatrix} + \begin{pmatrix} 21 \\ 10 \end{pmatrix} = \begin{pmatrix} 0 \\ -62 \end{pmatrix}$

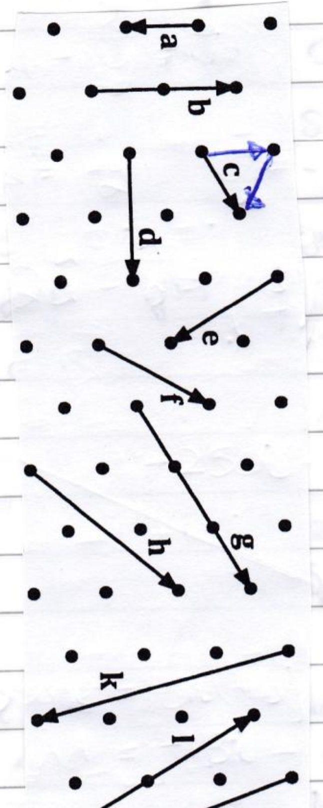
$\sqrt{62^2} = 62\,\text{m/sec}$

$\therefore t = \dfrac{300\,000}{62} = 4838.7\,\text{sec}$

$\therefore t \approx 1\,\text{hr}\ 21\,\text{min}$

17.





$\underset{\sim}{a} = -\underset{\sim}{p}$

$\underset{\sim}{b} = 2\underset{\sim}{p}$

$\underset{\sim}{c} = \underset{\sim}{p} + \underset{\sim}{q}$

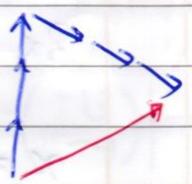
$\underset{\sim}{d} = \underset{\sim}{p} + 2\underset{\sim}{q}$

$\underset{\sim}{e} = -\underset{\sim}{p} + \underset{\sim}{q}$

$= \underset{\sim}{q} - \underset{\sim}{p}$

$\tilde{f} = 2\tilde{p} + \tilde{q}$

$\tilde{g} = 3\tilde{p} + 3\tilde{q}$



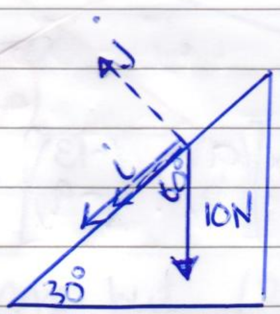
$\tilde{h} = 3\tilde{p} + 2\tilde{q}$

$\tilde{k} = -3\tilde{p} + \tilde{q}$

$\tilde{l} = 2\tilde{p} - 2\tilde{q}$

$\tilde{m} = -2\tilde{p} + \tilde{q}$

18.



$10\cos 60\, i - 10\sin 60\, j$

$= 5i - 5\sqrt{3}\, j$

19. $\tilde{a} = 2i + 3j \qquad \tilde{b} = i - j$

a) $x\tilde{a} + y\tilde{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$x\begin{pmatrix} 2 \\ 3 \end{pmatrix} + y\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

ie $2x + y = 3$ solve
$3x - y = 2$ simultaneous

$\therefore x = 1, y = 1.$

ie $\tilde{a} + \tilde{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

b) $x\begin{pmatrix} 2 \\ 3 \end{pmatrix} + y\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$

$2x + y = 5$
$3x - y = 5$
$x = 2 \qquad y = 1$

$\therefore 2\tilde{a} + \tilde{b} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$

c) $x\begin{pmatrix}2\\3\end{pmatrix} + y\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}1\\9\end{pmatrix}$

$$\left.\begin{aligned}2x + y &= 1\\ 3x - y &= 9\end{aligned}\right\}\text{solve}$$

$x = 2 \quad y = -3$

$\therefore\ 2\hat{a} - 3\hat{b} = \begin{pmatrix}1\\9\end{pmatrix}$

d) $x\begin{pmatrix}2\\3\end{pmatrix} + y\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}4\\7\end{pmatrix}$

$$\left.\begin{aligned}2x + y &= 4\\ 3x - y &= 7\end{aligned}\right\}\text{solve}$$

$x = \frac{11}{5} \quad y = -\frac{2}{5}$

$\frac{11}{5}\hat{a} - \frac{2}{5}\hat{b} = \begin{pmatrix}4\\7\end{pmatrix}$

e) $x\begin{pmatrix}2\\3\end{pmatrix} + y\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}3\\-1\end{pmatrix}$

$$\left.\begin{aligned}2x + y &= 3\\ 3x - y &= -1\end{aligned}\right\}\text{solve}$$

$x = \frac{2}{5} \quad y = \frac{11}{5}$

$\frac{2}{5}\hat{a} + \frac{11}{5}\hat{b} = \begin{pmatrix}3\\-1\end{pmatrix}$

f) $x\begin{pmatrix}2\\3\end{pmatrix} + y\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}3\\7\end{pmatrix}$

$$\left.\begin{aligned}2x + y &= 3\\ 3x - y &= 7\end{aligned}\right\}\text{solve}$$

$x = 2 \quad y = -1$

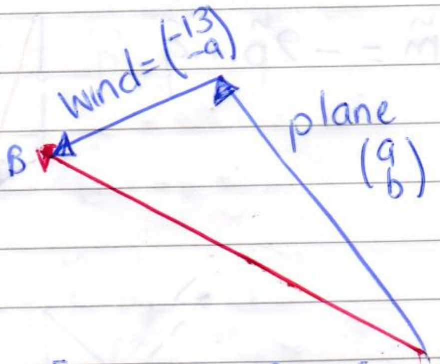
$2\hat{a} - \hat{b} = \begin{pmatrix}3\\7\end{pmatrix}$

20. 

$-250i + 750j$ km

plane $= ai + bj$
wind $= -13i - 9j$

$s = \frac{d}{t} \qquad s \times t = d$



$t \times \left[\begin{pmatrix}a\\b\end{pmatrix} + \begin{pmatrix}-13\\-9\end{pmatrix}\right] = \begin{pmatrix}-250\\750\end{pmatrix}$

$\begin{pmatrix}a\\b\end{pmatrix} + \begin{pmatrix}-13\\-9\end{pmatrix} = \frac{1}{t}\begin{pmatrix}-250\\750\end{pmatrix}$

$\begin{pmatrix}a\\b\end{pmatrix} = \frac{1}{t}\begin{pmatrix}-250\\750\end{pmatrix} - \begin{pmatrix}-13\\-9\end{pmatrix}$

note $|ai + bj| = 400$ km/hr

ie $\left|\frac{1}{t}\begin{pmatrix}-250\\750\end{pmatrix} - \begin{pmatrix}-13\\-9\end{pmatrix}\right| = 400$

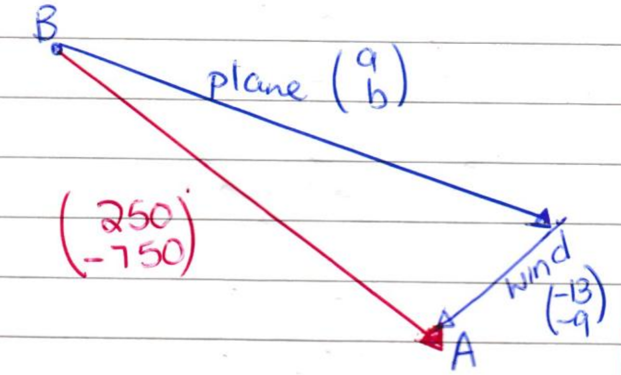
$\left(\frac{-250}{t} + 13\right)^2 + \left(\frac{750}{t} + 9\right)^2 = 400^2$

solve on calc. $t = 2$ or
$t = \cancel{-1.96}$ hr
cannot be -ve

ie $\begin{pmatrix}a\\b\end{pmatrix} = \frac{1}{2}\begin{pmatrix}-250\\750\end{pmatrix} - \begin{pmatrix}-13\\-9\end{pmatrix}$

$= (112i + 384j)$ km/hr

return journey



exact same but $\begin{pmatrix}250\\-750\end{pmatrix}$

t(plane + wind) = distance.

$\begin{pmatrix}a\\b\end{pmatrix} = \frac{1}{t}\begin{pmatrix}250\\-750\end{pmatrix} - \begin{pmatrix}-13\\-9\end{pmatrix}$

$\therefore \left|\frac{1}{t}\begin{pmatrix}250\\-750\end{pmatrix} - \begin{pmatrix}-13\\-9\end{pmatrix}\right| = 400$

$\left(\frac{250}{t} + 13\right)^2 + \left(\frac{-750}{t} + 9\right)^2 = 400^2$

$t = \cancel{2}$ $\quad t = 1.96$
cannot be -ve

$\therefore \begin{pmatrix}a\\b\end{pmatrix} = \frac{1}{1.96}\begin{pmatrix}250\\-750\end{pmatrix} - \begin{pmatrix}-13\\-9\end{pmatrix}$

$= (140.8i - 374.4j)$ km/hr

Ex 4C.

1.

$(2,5)$

$\overrightarrow{OA} = 2i + 5j$

$(-3, 6)$

$\overrightarrow{OB} = -3i + 6j$

$(0, -5)$

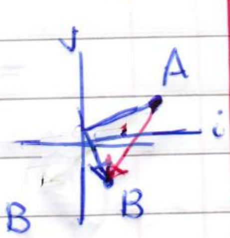
$\overrightarrow{OC} = 0i - 5j$

$= -5j$

$(3, 8)$

$\overrightarrow{OD} = 3i + 8j$

2. $OA = 3i + j \quad \overrightarrow{OB} = 2i - j$



$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= \begin{pmatrix} -3 \\ -1 \end{pmatrix} + \begin{pmatrix} +2 \\ -1 \end{pmatrix}$

$= i - 2j$

$\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA}$

$= \begin{pmatrix} -2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

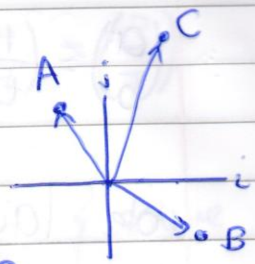
$= i + 2j$

* notice its just the opposite

3. $\overrightarrow{OA} = -i + 4j$

$\overrightarrow{OB} = 2i - 3j$

$\overrightarrow{OC} = i + 5j$



a) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= \begin{pmatrix} 1 \\ -4 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$= 3i - 7j$

b) $\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$

$= \begin{pmatrix} -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \end{pmatrix}$

$= -i + 8j$

c) $\overrightarrow{CA} = \begin{pmatrix} -1 \\ -5 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix}$

$= -2i - j$

4. $\overrightarrow{OA} = i + 2j$

$\overrightarrow{OB} = 4i - 2j$

$\overrightarrow{OC} = -i + 11j$

$\overrightarrow{OD} = 6i - 13j$

a) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= \begin{pmatrix} -1 \\ -2 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \end{pmatrix}$

$= 3i - 4j$

b) $\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$

$= \begin{pmatrix} -4 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ 11 \end{pmatrix} = \begin{pmatrix} -5 \\ 13 \end{pmatrix}$

c) $\vec{CD} = \vec{CO} + \vec{OD}$

$= \begin{pmatrix}1\\-11\end{pmatrix} + \begin{pmatrix}6\\-13\end{pmatrix}$

$= 7i - 24j$

d) magnitude of $|\vec{CD}|$

$= \sqrt{7^2 + 24^2} = 25$ units

Same direction as $\vec{AB}$ = unit vector

$|\vec{AB}| = \sqrt{3^2+4^2} = 5$

ie $\left(\frac{3i-4j}{5}\right) \times 25$

$= 15i - 20j$

5. $\vec{OA} = 3i + 7j$
$\vec{OB} = -2i + j$

a) $|\vec{OA}| = \sqrt{3^2+7^2}$
$= \sqrt{58}$ units

b) $|\vec{OB}| = \sqrt{2^2+1^2}$
$= \sqrt{5}$ units

c) $\vec{AB} = \vec{AO} + \vec{OB}$

$= \begin{pmatrix}-3\\-7\end{pmatrix} + \begin{pmatrix}-2\\1\end{pmatrix} = \begin{pmatrix}-5\\-6\end{pmatrix}$

$|\vec{AB}| = \sqrt{25+36} = \sqrt{61}$ units

6. $\vec{OA} = 2i + 3j$
$\vec{OB} = 5i - j$
$\vec{OC} = 3i + 7j$

a) $\vec{AB} = \vec{AO} + \vec{OB}$

$= \begin{pmatrix}-2\\-3\end{pmatrix} + \begin{pmatrix}5\\-1\end{pmatrix} = \begin{pmatrix}3\\-4\end{pmatrix}$

$|\vec{AB}| = \sqrt{9+16} = \sqrt{25} = 5$ units

b) $\vec{BA} = -\vec{AB} = \begin{pmatrix}-3\\4\end{pmatrix}$

$|\vec{BA}| = \sqrt{9+16} = 5$ units

c) $\vec{AC} = \vec{AO} + \vec{OC}$

$= \begin{pmatrix}-2\\-3\end{pmatrix} + \begin{pmatrix}3\\7\end{pmatrix} = \begin{pmatrix}1\\4\end{pmatrix}$

$|\vec{AC}| = \sqrt{1+16} = \sqrt{17}$ units

d) $\vec{BC} = \vec{BO} + \vec{OC}$

$= \begin{pmatrix}-5\\1\end{pmatrix} + \begin{pmatrix}3\\7\end{pmatrix} = \begin{pmatrix}-2\\8\end{pmatrix}$

$|\vec{BC}| = \sqrt{4+64} = \sqrt{68}$
$= 2\sqrt{17}$ units

7. $\vec{OA} = -i + 6j$
$\vec{OB} = 5i + 3j$

a) $|\vec{OA}| = \sqrt{1+36} = \sqrt{37}$ units

b) $|\vec{OB}| = \sqrt{25+9} = \sqrt{34}$ units

c) $\vec{AB} = \begin{pmatrix}1\\-6\end{pmatrix} + \begin{pmatrix}5\\3\end{pmatrix} = \begin{pmatrix}6\\-3\end{pmatrix}$

$|\vec{AB}| = \sqrt{6^2+3^2} = \sqrt{45} = 3\sqrt{5}$ units

8. $\vec{OA} = \begin{pmatrix}2\\-3\end{pmatrix}$ $\vec{OB} = \begin{pmatrix}1\\2\end{pmatrix}$

$\vec{OC} = \begin{pmatrix}9\\21\end{pmatrix}$ $\vec{OD} = \begin{pmatrix}6\\-2\end{pmatrix}$

a) $\vec{AB} = \vec{AO} + \vec{OB}$

$= \begin{pmatrix}-2\\3\end{pmatrix} + \begin{pmatrix}1\\2\end{pmatrix} = -i + 5j$

b) $\vec{BC} = \vec{BO} + \vec{OC}$

$= \begin{pmatrix}-1\\-2\end{pmatrix} + \begin{pmatrix}9\\21\end{pmatrix} = 8i + 19j$

c) $\vec{CD} = \vec{CO} + \vec{OD}$

$= \begin{pmatrix}-9\\-21\end{pmatrix} + \begin{pmatrix}6\\-2\end{pmatrix} = -3i - 23j$

d) $|\vec{AC}|$ ie $\vec{AC} = \vec{AO} + \vec{OC}$

$= \begin{pmatrix}-2\\3\end{pmatrix} + \begin{pmatrix}9\\21\end{pmatrix} = 7i + 24j$

$\sqrt{7 + 24^2} = 25$ units

e) $\vec{OA} + \vec{AB}$

$\begin{pmatrix}2\\-3\end{pmatrix} + \begin{pmatrix}-1\\5\end{pmatrix} = i + 2j$

f) $\vec{OA} + 2\vec{AC}$

$\begin{pmatrix}2\\-3\end{pmatrix} + 2\begin{pmatrix}7\\24\end{pmatrix} = 16i + 45j$

9. $\vec{OA} = 3i + 4j$

$\vec{AB} = 7i - j$

$\vec{AB} = \vec{AO} + \vec{OB}$

$\therefore \vec{OB} = \vec{AB} - \vec{AO}$

$= \begin{pmatrix}7\\-1\end{pmatrix} - \begin{pmatrix}-3\\-4\end{pmatrix}$

$= 10i + 3j$

10. $\vec{OA} = -i + 7j$

$\vec{AB} = 2i + 3j$

$\vec{AC} = 4i - 3j$

a) $\vec{OB}$?

$\vec{AB} = \vec{AO} + \vec{OB}$

$\therefore \vec{OB} = \vec{AB} - \vec{AO}$

$= \begin{pmatrix}2\\3\end{pmatrix} - \begin{pmatrix}1\\-7\end{pmatrix}$

$= i + 10j$

b) $\vec{AC} = \vec{AO} + \vec{OC}$

$\therefore \vec{OC} = \vec{AC} - \vec{AO}$

$= \begin{pmatrix}4\\-3\end{pmatrix} - \begin{pmatrix}1\\-7\end{pmatrix}$

$= 3i + 4j$

c) $\vec{BC} = \vec{BO} + \vec{OC} = \begin{pmatrix}-1\\-10\end{pmatrix} + \begin{pmatrix}3\\4\end{pmatrix}$

$= 2i - 6j$

11. $\vec{OA} = -i + 9j$
$\vec{OC} = 7i - j$

$\vec{BC} = 4i - 6j \quad \vec{DC} = 3i + 2j$

a) $\vec{BC} = \vec{BO} + \vec{OC}$
$\therefore \vec{BO} = \vec{BC} - \vec{OC}$
$= \begin{pmatrix} 4 \\ -6 \end{pmatrix} - \begin{pmatrix} 7 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$
$\therefore \vec{OB} = 3i + 5j$

b) $\vec{DC} = \vec{DO} + \vec{OC}$
$\therefore \vec{DO} = \vec{DC} - \vec{OC}$
$= \begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 7 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$
$\therefore \vec{OD} = 4i - 3j$

c) $\vec{BD} = \vec{BO} + \vec{OD}$
$= \begin{pmatrix} -3 \\ -5 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \end{pmatrix} = i - 8j$

d) $\vec{AD} = \vec{AO} + \vec{OD}$
$= \begin{pmatrix} 1 \\ -9 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ -12 \end{pmatrix}$
$|\vec{AD}| = \sqrt{25 + 144} = 13$ units

12. $\vec{OA} = 2i + 9j$
velocity $= (2i - 5j)$ m/sec

a) after 1 sec
$\begin{pmatrix} 2 \\ 9 \end{pmatrix} + 1\begin{pmatrix} 2 \\ -5 \end{pmatrix} = (4i + 4j)$ metres

b) after 2 sec
$\begin{pmatrix} 2 \\ 9 \end{pmatrix} + 2\begin{pmatrix} 2 \\ -5 \end{pmatrix} = 6i - j$ metres

c) after 10 sec
$\begin{pmatrix} 2 \\ 9 \end{pmatrix} + 10\begin{pmatrix} 2 \\ -5 \end{pmatrix} = 22i - 41j$ metres

d) after 5 sec
$\begin{pmatrix} 2 \\ 9 \end{pmatrix} + 5\begin{pmatrix} 2 \\ -5 \end{pmatrix} = 12i - 16j$

$|12i - 16j| = \sqrt{12^2 + 16^2}$
$= \sqrt{400}$
$= 20$ metres

13. $\vec{OA} = 5i - 6j$
velocity $= (i + 6j)$ m/sec

a) $t = 2$ sec
$\begin{pmatrix} 5 \\ -6 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 6 \end{pmatrix}$
$= (7i + 6j)$ metres.

b) $t = 3$ sec

$$\begin{pmatrix}5\\-6\end{pmatrix} + 3\begin{pmatrix}1\\6\end{pmatrix}$$

$= (8i + 12j)$ metres

c) $t = 7$ sec

$$\begin{pmatrix}5\\-6\end{pmatrix} + 7\begin{pmatrix}1\\6\end{pmatrix}$$

$= (12i + 36j)$ metres

d) $t = 5$

$$\begin{pmatrix}5\\-6\end{pmatrix} + 5\begin{pmatrix}1\\6\end{pmatrix} = \begin{pmatrix}10\\24\end{pmatrix}$$

$$|10i + 24j| = \sqrt{10^2 + 24^2}$$

$= 26$ metres

e) When will distance be 50m?

$$\left|\begin{pmatrix}5\\-6\end{pmatrix} + t\begin{pmatrix}1\\6\end{pmatrix}\right| = 50$$

$$(5+t)^2 + (-6+6t)^2 = 50^2$$

solve on calc.

$t = 9$ or $t = \cancel{\frac{-271}{37}}$

not -ve time

$\therefore$ $t = 9$ sec.

4. $\vec{OA} = 3i - j$

$\vec{OB} = -i + 15j$

$\vec{OC} = 9i - 25j$

collinear $\Rightarrow$ same direction

$\vec{AB} = k\,\vec{AC} = h\,\vec{BC}$

$\vec{AB} = \vec{AO} + \vec{OB}$

$= \begin{pmatrix}-3\\1\end{pmatrix} + \begin{pmatrix}-1\\15\end{pmatrix}$

$= -4i + 16j$

$\vec{AC} = \vec{AO} + \vec{OC}$

$= \begin{pmatrix}-3\\1\end{pmatrix} + \begin{pmatrix}9\\-25\end{pmatrix}$

$= 6i - 24j$

$\vec{BC} = \vec{BO} + \vec{OC}$

$= \begin{pmatrix}1\\-15\end{pmatrix} + \begin{pmatrix}9\\-25\end{pmatrix}$

$= 10i - 40j$

$-4i + 16j \Rightarrow -4(i - 4j)$

$6i - 24j \Rightarrow 6(i - 4j)$

$10i - 40j \Rightarrow 10(i - 4j)$

all have direction $i - 4j$.

$\therefore$ collinear.

15. $\vec{OD} = 9i - 7j \qquad \vec{OF} = 25i - 19j$

$OE = -11i + 8j$

for collinear.

$h\vec{DE} = k\vec{DF} = \lambda\vec{EF}$

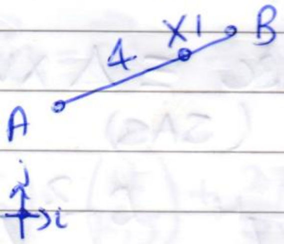
$\vec{DF} = \vec{DO} + \vec{OF} = \begin{pmatrix}-9\\7\end{pmatrix} + \begin{pmatrix}25\\-19\end{pmatrix} = \begin{pmatrix}16\\-12\end{pmatrix}$

$\vec{DE} = \vec{DO} + \vec{OE} = \begin{pmatrix}-9\\7\end{pmatrix} + \begin{pmatrix}-11\\8\end{pmatrix} = \begin{pmatrix}-20\\15\end{pmatrix}$

$\vec{EF} = \vec{EO} + \vec{OF} = \begin{pmatrix}11\\-8\end{pmatrix} + \begin{pmatrix}25\\-19\end{pmatrix} = \begin{pmatrix}36\\-27\end{pmatrix}$

$$\left.\begin{aligned}\vec{DF} &= 4(4i - 3j)\\ \vec{DE} &= -5(4i - 3j)\\ \vec{EF} &= 9(4i - 3j)\end{aligned}\right\} \text{ all } 4i - 3j \text{ ie collinear.}$$

16. $\vec{OA} = 2i + 5j \qquad \vec{OB} = 12i + 10j$



$\vec{AX} : \vec{XB}$
$4 : 1$

$\vec{AB} = \vec{AO} + \vec{OB} = \begin{pmatrix}-2\\-5\end{pmatrix} + \begin{pmatrix}12\\10\end{pmatrix} = \begin{pmatrix}10\\5\end{pmatrix}$

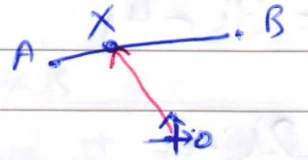
$\vec{OX} = \vec{OA} + \frac{4}{5}\vec{AB}$

$= \begin{pmatrix}2\\5\end{pmatrix} + \frac{4}{5}\begin{pmatrix}10\\5\end{pmatrix} = 10i + 9j$

17. $\vec{OA} = -2i + 2j \qquad \vec{OB} = 10i - j$

$\vec{AB}$ in the ratio 1:2

$\vec{AX} : \vec{XB} = 1:2$



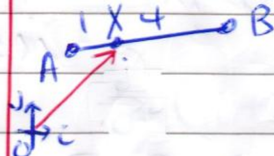
$\vec{AB} = \begin{pmatrix}2\\-2\end{pmatrix} + \begin{pmatrix}10\\-1\end{pmatrix} = \begin{pmatrix}12\\-3\end{pmatrix}$

$\vec{OX} = \vec{OA} + \frac{1}{3}\vec{AB}$

$= \begin{pmatrix}-2\\2\end{pmatrix} + \frac{1}{3}\begin{pmatrix}12\\-3\end{pmatrix}$

$= 2i + j$

18. $\vec{OA} = i + 8j \qquad \vec{OB} = 19i + 2j$



$\vec{AX} : \vec{XB}$
$1 : 4$

$\vec{AB} = \begin{pmatrix}-1\\-8\end{pmatrix} + \begin{pmatrix}19\\2\end{pmatrix} = \begin{pmatrix}18\\-6\end{pmatrix}$

$\vec{OX} = \vec{OA} + \frac{1}{5}\vec{AB}$

$= \begin{pmatrix}1\\8\end{pmatrix} + \frac{1}{5}\begin{pmatrix}18\\-6\end{pmatrix}$

$= 4.6i + 6.8j$

Misc Ex 4.

1. $\hat{a} = 2i + 3j$

$\hat{b} = 3i - 4j$

$\hat{c} = 2i + j$

$\hat{c} = \lambda\hat{a} + \mu\hat{b}$

$$\begin{pmatrix}2\\1\end{pmatrix} = \lambda\begin{pmatrix}2\\3\end{pmatrix} + \mu\begin{pmatrix}3\\-4\end{pmatrix}$$

ie $2 = 2\lambda + 3\mu$ & $1 = 3\lambda - 4\mu$

Solve simultaneously on calc

$2\lambda + 3\mu$ | μ, λ $\lambda = 11/17$

$3\lambda - 4\mu$ | $\mu = 4/17$

2. A B C D E F

2 or 3 letter codes, no repeats

$\underline{6} \times \underline{5} + \underline{6} \times \underline{5} \times \underline{4}$

$30 + 120 = 150$

3. If a positive whole n° ends in five (p) then the number is a multiple of five (q)

converse $q \Rightarrow p$

If a positive whole number ends in five, then it ends in a five.

contrapositive

$\bar{q} \to \bar{p}$

If a positive whole number is not a multiple of 5, then it does not end in a 5.

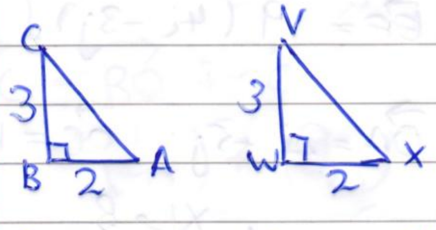
4. 5 careers to choose from a possible of 12

* must be in order

So permutation

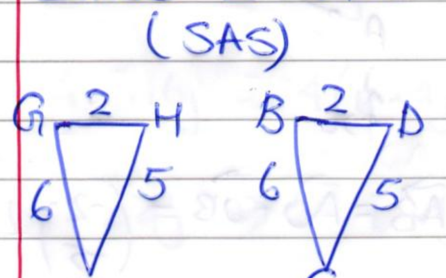
$12 \times 11 \times 10 \times 9 \times 8$

$= 95040$

5. 

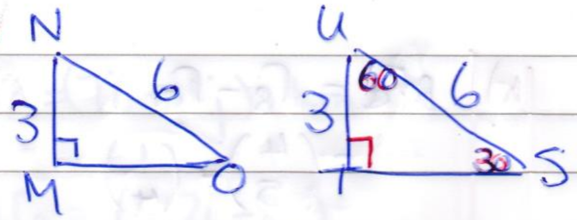
$\Delta ABC \equiv \Delta XWV$

(SAS)



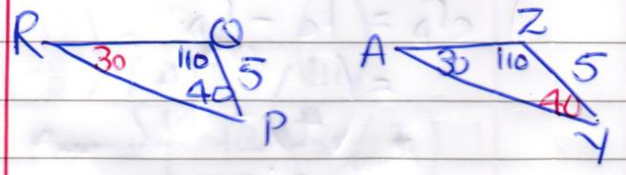
$\Delta GHI \equiv \Delta BDC$

(S.S.S)



$\Delta MNO \equiv \Delta TUS$

R.H.S.

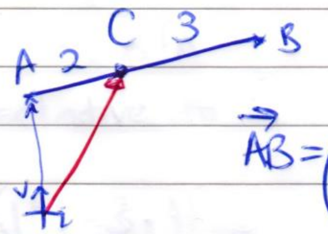


$\Delta RQP \equiv \Delta AZY$

(A.S.A)

6. $\vec{OA} = ai - 15j \quad \vec{OB} = 10i + bj$

$\vec{OC} = 4i - 3j \quad \vec{AC} : \vec{CB}$
$2 : 3$



$\vec{AB} = \begin{pmatrix} -a \\ 15 \end{pmatrix} + \begin{pmatrix} 10 \\ b \end{pmatrix}$

$\vec{OC} = \vec{OA} + \frac{2}{5}\vec{AB}$

$\begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} a \\ -15 \end{pmatrix} + \frac{2}{5}\begin{pmatrix} -a+10 \\ 15+b \end{pmatrix}$

ie $4 = a + \frac{2}{5}(-a+10)$

& $-3 = -15 + \frac{2}{5}(15+b)$

solve on calc

$a = 0 \qquad b = 15$

7. $n > 5000$

4 or 5 digits

using 1, 2, 3, 4, 5

a) no repeats

$\underset{\text{"5"}}{\underline{1}} \times \underline{4} \times \underline{3} \times \underline{2} + \underbrace{\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}}_{\text{all of these 5 digit n° are} > 5000}$

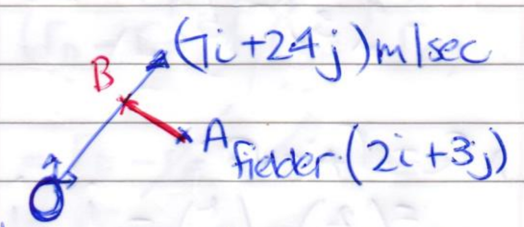
$= 144$

b) repeats allowed

$\underset{\text{"5"}}{\underline{1}}\ \underline{5} \times \underline{5} \times \underline{5} + \underline{5} \times \underline{5} \times \underline{5} \times \underline{5} \times 5$

$= 3250$

8.



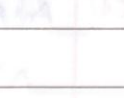
a) $|t(7i+24j)| = 60$

$(7t)^2 + (24t)^2 = 60^2$

solve $t = \cancel{-2.4}$ $t = 2.4$ sec

b) $|\vec{AB}| = \begin{pmatrix} -2 \\ -3 \end{pmatrix} + t\begin{pmatrix} 7 \\ 24 \end{pmatrix}$

$(7t-2)^2 + (24t-3)^2$



graph & find min

at $t = 0.1376$ sec

$\sqrt{1.1664} = 1.08$ m